



MATHEMATICS APPLICATIONS

Calculator-free

ATAR course examination 2022

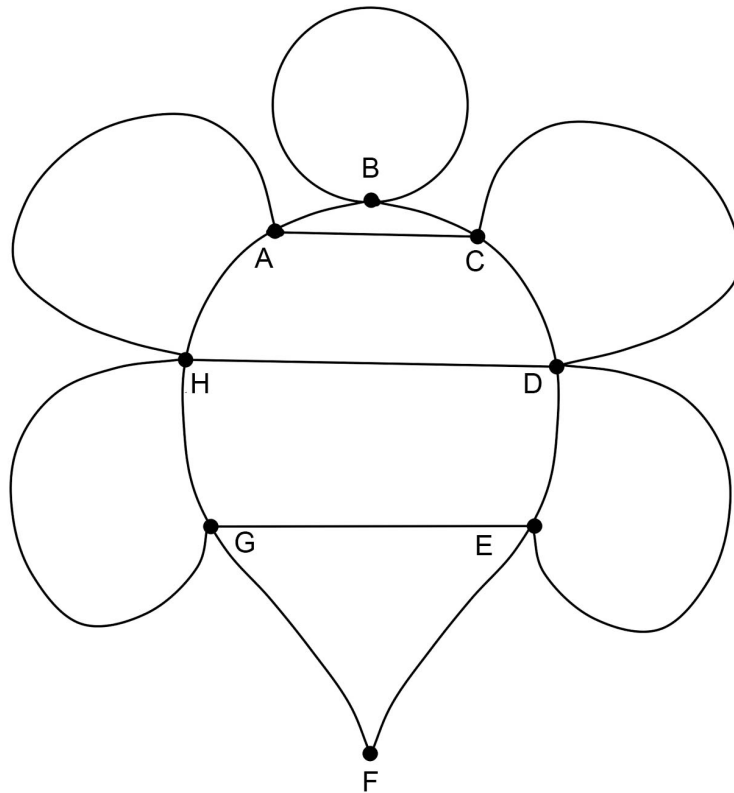
Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

Question 1

(7 marks)

The network below shows the different paths around a botanical exhibition.



- (a) Explain why this is **not** a simple graph. (1 mark)

Solution
This network is not a simple graph as it contains loops or multiple edges
Specific behaviours
✓ states a correct reason

- (b) Is this network Eulerian or semi-Eulerian? Justify your response. (2 marks)

Solution
This network is semi-Eulerian because there are exactly two odd vertices (D and H have degree of 5).
Specific behaviours
✓ states network is semi-Eulerian
✓ Identifies two odd vertices

- (c) Use Euler's formula to prove that this network is planar. (2 marks)

Solution
$f + v - e = 2$ $10 + 8 - 16 = 2$ $LHS = RHS \therefore$ planar
Specific behaviours
✓ states correct number of faces, vertices and edges ✓ correctly shows $LHS = RHS$

- (d) A visitor to the exhibition has taken her child in a pram and wishes to walk along every path exactly once to see all the floral displays, excluding the walk back to the car park. If the only car parks are located next to D and F, where would you suggest they park? Use mathematical reasoning to support your answer. (2 marks)

Solution
Network is semi-Eulerian. Suggestion would be to park near start vertex D \therefore car park near D.
Specific behaviours
✓ states car park D ✓ provides valid reason

Question 2

(7 marks)

A gardener purchases a new lawnmower valued at \$4800. The lawnmower depreciates at a constant rate of \$250 per year.

- (a) (i) Determine a recursive rule for the value of the lawnmower after n years. (2 marks)

Solution
$T_{n+1} = T_n - 250, T_0 = 4800$
Specific behaviours
✓ states correct rule ✓ states $T_0 = 4800$

- (ii) Deduce a rule for the n^{th} term of this sequence. (1 mark)

Solution
$T_n = 4800 - 250n$
Specific behaviours
✓ states correct rule

- (b) Determine the value of the lawnmower after 4 years. (1 mark)

Solution
$T_4 = 4800 - 250(4)$ $= \$3800$
Specific behaviours
✓ determines correct value

As part of his business plan, the gardener will sell his lawnmower when its value drops below \$1300.

- (c) Calculate when the lawnmower will be sold. (3 marks)

Solution
$1300 = 4800 - 250n$ $250n = 3500$ $n = 14 \therefore$ after 14 years
Specific behaviours
✓ identifies difference of 3500 ✓ correctly calculates the value of n ✓ correctly states after 14 years

Question 3

(8 marks)

An analysis was done on hospital patients based on their age and injuries received from various outdoor activities. The information is displayed below.

		Activity			Total
		Trampoline	Bike	Skateboard	
Age (years)	0–8	150	120	30	300
	9–16	180	180	90	450
	17–24	120	80	200	400
	25–32	20	150	80	250
	Total	470	530	400	1400

- (a) Complete the two-way table above. (2 marks)

Solution	
See table above	
Specific behaviours	
✓ correctly completes three or more entries	
✓ correctly completes all entries	

- (b) Identify the explanatory variable for these data. (1 mark)

Solution	
Age	
Specific behaviours	
✓ correctly identifies age	

- (c) The incomplete two-way percentaged table is shown below.

		Activity			Total
		Trampoline	Bike	Skateboard	
Age (years)	0–8	50	40	10	100
	9–16	40	40	20	100
	17–24	30	20	50	100
	25–32	8	60	32	100
	Total				

- (i) Show how the value of 20% was calculated. (1 mark)

Solution	
$\frac{90}{450} = \frac{1}{5} = 20\%$	
Specific behaviours	
✓ shows correct calculation	

Question 3 (continued)

- (ii) Complete the table by using either row percentages **or** column percentages as appropriate. (3 marks)

Solution
See above table
Specific behaviours
✓ correctly identifies row percentages required and row totals equal 100
✓ correctly calculates values for age 25–32
✓ correctly calculates remaining values

- (d) State an association that can be observed from the two-way percentaged table. (1 mark)

Solution
As a patient’s age increases, injuries received due to trampoline decreases.
Specific behaviours
✓ correctly states association
Accept other relevant answers.

Question 4

(9 marks)

The project network shown below is to be followed when installing a new conveyor belt system for a mining company. The tasks, with their completion time in days, are shown on the network. The project will commence on 20 November, with work continuing for seven days each week.

(a) Determine the critical path and the minimum completion time. (2 marks)

Solution	
DEFNP 33 days	
Specific behaviours	
✓ correct path ✓ correct length	

(b) Determine the immediate predecessors for Task P. (2 marks)

Solution
M, N, K, L and J
Specific behaviours
✓ at least 3 correct predecessors ✓ all correct predecessors

(c) What is the latest date in November that Task H can commence? (2 marks)

Solution
$33 - (8 + 4 + 11) = 10$ days Latest start date is 30 November
Specific behaviours
✓ correctly determines number from start of the project ✓ correctly determines correct date

Question 4 (continued)

- (d) It has been confirmed that there is a delay in obtaining some parts from overseas. This means Task G will take 5 days longer to complete. Discuss all implications for the completion of the project. A copy of the **original** network is given below. (3 marks)

Solution	
<p>The diagram shows a project network with the following tasks and durations: A(7), B(4), C(11), D(6), E(7), F(9), G(8), H(8), J(12), K(6), L(4), M(5), N(6), P(5), Q(11), R(4). The original critical path is DEGLQ (36 days). The new critical path after a 5-day delay in task G is DEGLQ (36 days).</p>	
<p>New critical path is: DEGLQ New completion time is: 36 days</p>	
Specific behaviours	
<ul style="list-style-type: none"> ✓ correctly indicates change to task G ✓ correctly states the new critical path ✓ correctly states the new completion time 	

Question 5

(12 marks)

A stall at a local market sells leather handbags. Sales over the past three weeks are tabulated, along with other calculations.

Week	Day	<i>t</i>	Sales	3 point moving average	Daily mean	Percentage of daily mean	Deseasonalised sales
1	Friday	1	2	—	5	40	4
	Saturday	2	9	5		180	6
	Sunday	3	4	5.33		80	4
2	Friday	4	3	4.33	5	60	6
	Saturday	5	6	5		120	4
	Sunday	6	6	5		120	6
3	Friday	7	3	6	6	50	6
	Saturday	8	9	6		150	
	Sunday	9	6	—		100	

(a) Show clearly how each of the numbers in bold type was calculated. (3 marks)

Solution	
5.33:	$\frac{9+4+3}{3} = \frac{16}{3} = 5.33$
120:	$\frac{6}{5} \times 100 = 120$
6:	$\frac{3+9+6}{3} = \frac{18}{3} = 6$
Specific behaviours	
<ul style="list-style-type: none"> ✓ correctly shows addition $9 + 4 + 3$ and division by 3 ✓ correctly shows $6 \div 5$ and conversion to a percentage ✓ correctly shows addition $3 + 9 + 6$ and division by 3 	

(b) Determine the seasonal index for Friday and explain its meaning. (2 marks)

Solution	
	$\frac{40+60+50}{3} = \frac{150}{3} = 50$
On average, Friday values are 50% of the daily mean	
Specific behaviours	
<ul style="list-style-type: none"> ✓ correctly calculates 50 ✓ states the link to daily mean 	

Question 5 (continued)

- (c) (i) Give a reason why time series data are deseasonalised. (1 mark)

Solution
To remove the seasonal effect
Specific behaviours
✓ states a correct reason

- (ii) Calculate the deseasonalised value for Friday of Week 3. (2 marks)

Solution
$3 \div 50\% = 3 \div 0.5 = 6$
Specific behaviours
✓ correctly shows division by 50% or 0.5
✓ correctly calculates the value of 6

The equation of the least-squares line for the deseasonalised sales, based on time t , is $y = 0.2t + 4.3$.

- (d) How does this equation support the observation that sales are increasing? (1 mark)

Solution
Positive gradient on a straight line means the values are increasing
Specific behaviours
✓ correctly identifies the positive gradient

- (e) Predict the sales for Friday of Week 4. (3 marks)

Solution
$y(t = 10) = 0.2 \times 10 + 4.3 = 6.3$
$6.3 \times 50\% = 3.15$
Rounded to 3 handbags
Specific behaviours
✓ correctly substitutes $t = 10$
✓ correctly complete calculation
✓ gives the correct rounded value of 3

Question 6

(8 marks)

A company has four different models of robotic welders that are used to assemble components that require welding. Each of these welders must be set up to assemble just one type of component. The number of components that each welder can assemble per hour for the three different types of components required is shown below.

		Welders			
		A	B	C	D
Components	1	15	12	13	11
	2	13	16	13	9
	3	15	16	12	14

- (a) Use the Hungarian Algorithm to determine the allocation of welder to component that will maximise the number of components assembled. (6 marks)

Solution	
$\begin{bmatrix} 15 & 12 & 13 & 11 \\ 13 & 16 & 13 & 9 \\ 15 & 16 & 12 & 14 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	
$\begin{bmatrix} 1 & 4 & 3 & 5 \\ 3 & 0 & 3 & 7 \\ 1 & 0 & 4 & 2 \\ 16 & 16 & 16 & 16 \end{bmatrix}$	
$\begin{bmatrix} 0 & 3 & 2 & 4 \\ 3 & 0 & 3 & 7 \\ 1 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	
$\begin{bmatrix} 0 & 3 & 2 & 4 \\ 3 & 0 & 3 & 7 \\ 1 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	
$\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 5 \\ 1 & 0 & 2 & 0 \\ 2 & 2 & 0 & 0 \end{bmatrix}$	
Allocation: A – 1, B – 2, D – 3	
Specific behaviours	
<ul style="list-style-type: none"> ✓ shows square matrix with row of zeros ✓ shows subtraction of each score from 16 ✓ shows subtraction of lowest score in each row from each score in that row ✓ shows all zeros can be covered by just 3 lines ✓ shows subtraction of 2 from each uncovered score and 2 added to each score covered by 2 lines ✓ determines correct allocation 	

Question 6 (continued)

- (b) State how many of each component will be produced each hour with the allocation determined in part (a). (2 marks)

Solution
Component 1 – 15 Component 2 – 16 Component 3 – 14
Specific behaviours
✓ three correct values ✓ all values linked to correct components

Question 7

(3 marks)

The following table shows activities and their immediate predecessors for a project to develop, produce and sell a new interactive game for children.

Activity	Activity description	Immediate Predecessor
A	Market analysis	-
B	Product design	A
C	Manufacture product	B
D	Develop software	B
E	Software testing	C, D
F	Design and produce packaging	C
G	Assemble product	E, F
H	Sales promotion	G

Draw the project network for the information given in the table.

Solution
<pre> graph LR Start(()) -- A --> B(()) B -- B --> C(()) B -- B --> D(()) C -- C --> E(()) D -- D --> E E -- E --> F(()) C -- F --> F F -- F --> G(()) E -- E --> G G -- G --> H(()) </pre>
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly shows direction of flow throughout the network ✓ correctly shows at least 6 correct activity positions ✓ correctly shows all activities

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